Article: Derivation of the frequencies of the 12 notes

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A Simple Derivation of the Frequencies of the 12 notes

1. Introduction

This amazingly simple experiment uses:

(i) An instrument which can measure frequency of a musical sound -- A 'frequency meter'. Let us say that frequency of a tone is the same as its pitch. This meter reports the frequency of a tone in units calles Hertz, Hz for short.

(ii) the fourth string of the tanpura: this is the lowermost-sounding thick copper string. It should sound firm and strong. It is not required to be tuned in any particular frequency. Call its pitch our Sa (S).

You are the experimenter. You have good, sensitive ear for music.

2. The Experiment

Now, you have plucked the tanpura string mentioned above. First, you hear the loud, strong, easily audible note. This is the note that we have called our Sa, the fundamental tone. But wait ... your sensitive ears also pick up two more easy-to-grasp, clear, ringing notes: Pa and Ga. You can hear more notes, too. But we are concerned here with only the three notes: Sa, Pa, and Ga. The Sa is the plucked note; Pa and Ga are self-generated ('swayam-bhoo' in Sanskrit) when you pluck the string.

[There are many self-generated tones heard from this very string, like Sa in other octaves, Pa in other octaves, Ga in other octaves, Re. komal Ni, etc. All these self-generated tones form the Harmonic Series. Our Sa is called the fundamental, while other self-generated notes are called overtones or partials. Let us briefly refer to the first five partials: the first is the Sa, the fundamental; the second partial is also Sa, but one octave up; the third is Pa, also one octave up from the fundamental Sa; the fourth partial is Sa again, but two octaves up; and the fifth partial is Ga, two octaves up compared to the fundamental Sa.]

Now, with the help of the 'frequency meter', mentioned above, measure the frequencies of the Sa, Pa and Ga. Since these three notes are in different octaves, bring them to the middle octave by a simple calculation (with which we will not concern ourselves as yet!).

The frequency meter shows the ratios of the Sa, Ga, and Pa as under:

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Sa:Ga:Pa
4 :5 :6
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Thus, for example, is you happen to have tuned your Tanpura string (which we have taken as our Sa) such that the frequency meter shows it to be vibrating at a frequency of 100 Hz, then the meter will show the self-generated Ga to be at 125 Hz, and Pa at 150 Hz.

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Sa :Ga :Pa
100:125:150
4  :5  :6 (dividing each by 25).
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Or, to give another example, if the frequency meter shows our Sa to be tuned at 240 Hz, it will show the self-generated Ga to be at 300 Hz, and Pa at 360 Hz. Again, Sa :Ga :Pa 240:300:360 4 :5 :6 (dividing each by 60).

3. The Calculations

Using some primary-school-level arithmatic, let us do some simple calculations. These calculations result in the formation of the natural, or Just Scale. There are twelve notes to an octave, namely Sa (S), komal Re (r), shuddha Re (R), komal Ga (g), shuddha Ga (G), shuddha Ma (m), teevra Ma (M), Pa (P), komal Dha (d), shuddha Dha (D), komal Ni (n), and shudha Ni (N). Out of these, we know the frequencies of only three notes as a result of the experiment described above. The ratios of the frequencies of these three notes are: 4:5:6. You must have noticed that, whatever the frequencies of those notes, their frequency ratios remain 4:5:6. So, ratios, also called intervals, are more fundamental than frequencies measured in units of Hz.

3A. Finding the frequencies of Re, and Ni (both shuddha):

Pa, Ni and Re stand in the same relationship as Sa, Ga and Pa. We know the frequency of Pa to be 360 in the above experiment. Hence Sa:Ga:Pa :: Pa:Ni:Re :: 4:5:6 . Since Pa = 360 Hz = 4 X 90, Ni = 5 X 90 = 450 Hz; and Re = 6 X 90 = 540 Hz.

Her we will see how to calculate "octave-reduction". For proper comparison, we need to have frequencies in the same octave -- call it the middle octave. If Sa is 240 Hz, the Sa of the next octave (S') has the frequency of 2 X 240 = 480 Hz. If the frequency of any note is greater than 480, it belongs to the next octave, and needs to be brought down to the middle octave. In our present example, the frequencies of all the notes must lie between 240 Hz and 480 Hz. The Re above has the frequency of 540 Hz. Divide it by 2, i.e. 540/2 = 270 Hz.

So, now we have the frequencies of the following notes: Sa 240 Hz, Re 270 Hz, Ga 300 Hz, Pa 360 Hz, and Ni 450 Hz.

3B. Finding the frequencies of shuddha Ma, and shuddha Dha:

Let us assume, for a moment, that shuddha Ma in the middle octave is our Sa. Here, remember that the Sa in the next octave (S') has the frequency of 2 X 240 = 480 Hz. Ma, Dha and Sa' stand in the same relationship as Sa, Ga and Pa. We know the frequency of Sa'to be 480 Hz. Here we are using the Sa of the next octave to get the value of Ma and Dha in the middle octave.

Ma:Dha:Sa' :: Sa:Ga:Pa :: 4:5:6 and Sa (S') is 480 Hz. Divide 480 by 6, we get 80. Hence multiply 4 by 80 to get the frequency of Ma. Multiply 5 by 80 to get the frequency of Dha. Thus, we get

Ma:Dha:Sa' :: 80 X 4:80 X 5:80 X 6: 80 = 320:400:480. Hence the frequency of Ma is 320 Hz, the frequency of Dha is 400 Hz.

So, now we have the frequencies of the following notes: Sa 240 Hz, Re 270 Hz, Ga 300 Hz, Ma 320 Hz,Pa 360 Hz, Dha 400 Hz, and Ni 450 Hz. So, now we know the frequencies of all the seven shuddha notes.

3C. Finding the frequency of komal Ga (g):

Ga (G), Pa (P) and Ni (N) stand in the same relationship as Sa, komal Ga and Pa. It follows that G:P:: S:g :: 300:360 = 5/6. So, g = 6*240/5 = 288 Hz. Hence komal Ga (g) has the frequency of 288 Hz.

3D. Finding the frequency of komal Ni (n):

Komal Ga (g) and komal Ni (n) stand in the same relationship as Sa and Pa.

Hence, g/n = S/P = 2/3. We already know the frequency of komal Ga (g) to be 288 Hz from above. Hence komal Ni (n) has the frequency of 3/2 = 288/n. So, n = 3*288/2 = 432 Hz.

3E. Finding the frequency of Teevra Ma (M):

Shuddha Ni (N) and teevra Ma (M) stand in the same relationship as Sa and Pa.

N:M' :: S:P :: 2:3 :: 450:M'. So, M' = 3*450/2 = 675 Hz. Hence teevra Ma' (M') of the next octave has the frequency of 675 Hz. So teevra Ma (M) in the middle octave has the frequency of 675/2 = 337.5 Hz.

To sum up, the frequencies of the twelve notes of an octave are as follows (in Hz) :-

S 240, r 256, R 270, g 288, G 300, m 320, M 337.5, P 360, d 384, D 400, n 432, N 450. The Sa of the next octave (S') will have a frequency of 480 Hz.

3F. Finding the frequency of komal Re (r):

Komal Re (r) and shuddha Ma (m) stand in the same relationship as Sa and Ga.

Hence, r/m = S/G = 4/5. We already know the frequency of komal Ma (m) to be 320 Hz. So, r/320 = 4/5. Hence r = 4*320/5 = 256 Hz.

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